FIXED-POINT IMPLEMENTATION OF TWO-CHANNEL IIR FILTER BANKS WITH VARIABLE CROSSOVER FREQUENCY

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ABSTRACT
A simple tuning scheme that shifts the crossover frequency of the start-up IIR half-band filter pair to the desired location has been introduced recently. In this paper, the approximation of the tuning scheme is derived in a manner that makes this scheme suitable for fixed-point arithmetic. The effects of the approximation error to the frequency responses and to the complementary properties for two classes of two-channel filter banks are analyzed. Class I denotes the IIR filter pair implemented as a parallel connection of two all-pass filters, whereas Class II stays for the tapped cascaded interconnection of two identical all-pass filters. Real-time application of the proposed method is demonstrated on the example filter pairs for fixed-point signal processor implementation.

1. INTRODUCTION
Complementary filter pairs with variable characteristics are of interest in applications where there is a need to change some of spectral characteristics during the course of signal processing. Variable digital filters are important in many applications, such as telecommunications, digital audio, laboratory instrumentations, and others. Implementing variable digital filter means that new filter constants should be computed in the simplest possible manner using constants of the existing filter. Due to high sensitivity to filter constants, design of an IIR variable filter-pair is more difficult than design of a variable FIR filter-pair. For the review of variable digital filter methods see eg. [1].

For the IIR filter implementation based on the parallel connection of two all-pass filters, an efficient implementation of the variable cutoff has already been developed and shown in [2] and [3]. The tuning of the filter constants in [2] and [3] was performed gradually using the approximation of the classical low-pass-to-low-pass transformation given in [4].

In this paper, we consider two classes of IIR complementary filter pairs: the term Class I stays for the implementation structure based on the parallel connection of two all-pass filters, whereas Class II stays for the tapped cascaded interconnection for two identical all-pass filters. For the tuning of the cutoff frequency we use the tuning formulae derived in [5], [6]. It was shown in [5] and [6] that, starting from an elliptic or Butterworth half-band filter-pair, the new filter pair with the cutoff frequency located at the desired position could be obtained in the single step by using extremely simple formulae. The new filter-pair has the pass- stop-band characteristics identical with those of the start-up half-band filter pair. In this paper, we examine the practical aspects of the implementation of variable digital filter pairs first introduced in [5], [6]. We apply the approximation of tuning formulae first introduced in [7] to perform tuning of the crossover frequency of a complementary filter pair in fixed-point arithmetic. We also examine the signal processor implementation of those two classes of variable two-channel filter banks.

This paper is divided in seven major sections. After the introduction, section 2 presents the review of Class I complementary filter pairs based on the parallel connection of two all-pass filters. The simple tuning formulae introduced in [5] and [6] are given, and the accuracy problem of the fixed-point implementation is discussed using the sensitivity analysis. Section 3 presents the low-sensitivity implementation structure for the Class II complementary filter pairs, which is based on the tapped cascaded interconnection of two identical all-pass filters. Section 4 introduces the approximation of tuning formulae suitable for fixed-point implementation. In section 5, the signal processor implementation is described. In section 6, the experimental results for two classes of complementary filter pairs implemented in fixed-point signal processor are displayed and compared. Section 7 is the conclusion.

2. CLASS I IIR FILTER PAIRS
This section presents properties of Class I digital filter pairs used in the paper. First, the filter transfer function is an elliptic minimal Q-factor (EMQF) filter introduced in [8], [9] or a Butterworth filter. Common property of these transfer functions is that their z-plane poles are placed on the circle orthogonal with the unit circle and
centered on the real axis. An elliptic half-band filter, or Butterworth half-band filter are just special cases. Secondly, the implementation structure is based on the parallel connection of two all-pass filters.

2.1. Implementation Structure

An odd-order low-pass/high-pass filter pair can be implemented as a parallel connection of two all-pass functions $A_{d}(z)$ and $A_{i}(z)$. The low-pass and high-pass transfer functions $G_{LP}(z)$ and $G_{HP}(z)$, are expressible by

$$ G_{LP}(z) = \frac{A_{d}(z)}{2} + \frac{A_{i}(z)}{2} $$

$$ G_{HP}(z) = \frac{A_{d}(z)}{2} - \frac{A_{i}(z)}{2} $$

Fig. 1 depicts an efficient implementation of equations (1a) and (1b).

![Figure 1](Image)

This filter pair exhibits all-pass complementary and power complementary properties.

The all-pass functions $A_{d}(z)$ and $A_{i}(z)$ can be expressed in the product form, and implemented as a cascade connection of the first order section, which implements the real filter pole, and the second order sections, each of them implementing the pair of conjugate complex poles. Hence, $A_{d}(z)$ and $A_{i}(z)$ are represented in the form,

$$ A_{d}(z) = \frac{\alpha_{i} + z^{-1} \prod_{i=1}^{N/2} \beta_{i} + \gamma_{i} z^{-i} + z^{-2}}{1 + \alpha_{d} z^{-1} + \gamma_{d} + \beta_{d} z^{-2}} $$

$$ A_{i}(z) = \prod_{i=2,4}^{N/2} \beta_{i} + \gamma_{i} z^{-i} + z^{-2} $$

In many applications, an alternative form for the second-order all-pass sections in $A_{d}(z)$ and $A_{i}(z)$ is used,

$$ A_{d}(z) = \frac{\alpha_{i} + z^{-1} \prod_{i=1}^{N/2} \beta_{i} + \alpha(1 + \beta_{i}) z^{-i} + z^{-2}}{1 + \alpha_{d} z^{-1} + \alpha(1 + \beta_{d}) z^{-2} + \beta_{d} z^{-2}} $$

$$ A_{i}(z) = \prod_{i=2,4}^{N/2} \beta_{i} + \alpha(1 + \beta_{i}) z^{-i} + z^{-2} $$

Here, $N$ is the filter order, an odd number, $\alpha$ represents the real pole, and $\beta_{i}$, $i=2,3,\ldots,(N-1)/2$ are the constants determined by the radii $r_{i}$ of the conjugate complex pole pairs, i.e.,

$$ \beta_{i} = (r_{i})^{2}, \text{ with } \beta_{i} < \beta_{i+1}. $$

Note that in (3a) and (3b) all the second-order sections share the constant $\alpha$ which is of the same value for all the second-order sections. This is a common property of EMQF and Butterworth filters.

In the case of a half-band filter (whose poles are placed on the imaginary axis as it is the case for an elliptic or Butterworth half-band filter) the all-pass filters $A_{d}^{m}(z)$ and $A_{i}^{m}(z)$ are expressible in the product form as follows,

$$ A_{d}^{m}(z) = \prod_{i=1}^{N/2} \frac{\beta_{i}^{m} + z^{-2}}{1 + \beta_{i}^{m} z^{-2}} $$

$$ A_{i}^{m}(z) = \prod_{i=2}^{N/2} \beta_{i}^{m} + z^{-2} $$

with

$$ \beta_{i}^{m} = (r_{i})^{2}, \beta_{i+1}^{m} < \beta_{i+2}^{m}. $$

The term $z^{-1}$ in (4a) represents the half-band filter first-order section, which implements the pole placed at the origin.

It was derived in [5], [6] that the 3-dB cutoff frequency of a half-band filter could be moved from its location $\omega_{0}^{m} = \pi / 2$ to any desired position in the range $0 < \omega < \pi$. In this way, from a start-up half-band filter pair $[G_{LP}(z), G_{HP}(z)]$ the new filter pair $[G_{LP}(z), G_{HP}(z)]$ that retains the pass- stop-band characteristics of the start-up half-band filter pair is obtained. Thereby, if the start-up filter pair $[G_{LP}(z), G_{HP}(z)]$ is an elliptic (Butterworth) filter, the new filter pair $[G_{LP}(z), G_{HP}(z)]$ is also an elliptic (Butterworth) filter. Moreover, since an elliptic half-band filter belongs to the class of EMQF filters, an elliptic $G_{LP}(z)$ $[G_{HP}(z)]$ should be an EMQF filter. Note that the 3dB-cutoff in $G_{LP}(z) [G_{HP}(z)]$ is the crossover frequency for the complementary filter pair $[G_{LP}(z), G_{HP}(z)]$.

2.2. Frequency Transformations

This subsection presents the simple tuning formulea derived in [5], [6] that compute the constants of $G_{LP}(z)$ and $G_{HP}(z)$ from those of $G_{LP}(z)$ and $G_{LP}(z)$ and the desired 3-dB cutoff $\omega_{0}$.

The resulting tuning formulae are based on the low-pass-to-low-pass frequency transformation. The well-known transform [4], $Q(z) = (z^{-1} - g)/(1 - g z^{-1})$, with $g = \sin[\pi / 2 - \omega_{0}]/2/\sin[\pi / 2 + \omega_{0}]/2$, is applied to $G_{LP}^{m}(z)$ $[G_{HP}^{m}(z)]$ using the replacement $z^{-1} \Rightarrow Q(z)$. This way, the constants of (3a) and (3b) can be expressed in terms of the constants of (4a) and (4b), and the desired value of $\omega_{0}$. Applying the above transformations directly to the first-order and second-order all-pass
sections, the following expressions are obtained for the constants $\alpha$, $\alpha_1$ and $\beta_i$ (see [5] and [6]):

$$\alpha = -\cos(\omega_i),$$  \hspace{1cm}(5)$$

$$\alpha_1 = \frac{1}{\alpha}(1 - \sqrt{1 - \alpha^2}),$$ \hspace{1cm}(6)$$

$$\beta_i = \left(\beta_i^{1.0} + \alpha_1^2\right)\left(\beta_i^{2.0} + 1\right), \hspace{0.5cm} i = 2,3,\ldots,(N+1)/2 \hspace{1cm}(7)$$

Using simple formulae given in (5), (6) and (7), the constants of the new filter are computed in a single step. The objective of this paper is to consider the fixed-point implementation of those tuning formulae that will be discussed in sequel.

### 2.3. Sensitivity Analysis

Fixed-point implementation introduces the quantization error in tuning formulae (5), (6) and (7), and consequently, new constants of the transformed filter pair are implemented with some error. The sensitivity analysis of the amplitude response shows the effects of the quantization error to the accuracy of the proposed tuning procedure.

It is well known [9], [10] that the amplitude responses of a low-pass/high-pass filter implemented according to (1a) and (1b) and Fig. 1 is expressible by

$$A_{LP}(\omega) = \cos(\omega), \hspace{1cm} A_{HP}(\omega) = \sin(\omega) \hspace{1cm}(8)$$

where $A_{LP}(\omega)$ [$A_{HP}(\omega)$] stays for the amplitude response of low-pass [high-pass] filter, and $\psi(\omega)$ is the phase difference of the all-pass branches $A_{LP}(z)$ and $A_{HP}(z)$,

$$\psi(\omega) = \frac{\phi_0(\omega) - \phi_1(\omega)}{2} \hspace{1cm}(9)$$

It was shown [9] that the sensitivity function for the low-pass filter is given by

$$S_a^L(\omega) = \pm \sin(\omega) \times \frac{\partial \psi(x,\omega)}{\partial x} \hspace{1cm}(10)$$

and similarly for high-pass filter

$$S_a^H(\omega) = \pm \cos(\omega) \times \frac{\partial \psi(x,\omega)}{\partial x} \hspace{1cm}(11)$$

The term $\partial \psi(x,\omega)/\partial x$ is the phase sensitivity of the first-order or second-order all-pass section in which the constant $x$ resides. In (10) and (11), the term $\partial \psi(x,\omega)/\partial x$ is suppressed in the pass bands since $\sin(\omega)$ for low-pass filter [$\cos(\psi(x,\omega))$ for high-pass filter] approximates zero in the pass-band. Consequently, the pass-band sensitivity is very small. In stop-band, $\sin(\omega)$ for low-pass filter [$\cos(\psi(x,\omega))$ for high-pass filter] approximates unity and the stop-band sensitivity is nearly equal to the term $\partial \psi(x,\omega)/\partial x$. Thereby, the phase sensitivity of the individual sections mainly determines the maximal sensitivity of the amplitude response.

Next, we illustrate the phase sensitivities $\partial \psi(x,\omega)/\partial x$ to all filter constants in (2) and (3). We choose example 7th order and 3rd order filters having equal transition bands, and for those examples we plot the phase sensitivities $\partial \psi(x,\omega)/\partial x$ for the start-up half-band filters, $\omega_c = 0.5\pi$, and for the filters with the cutoffs at $\omega_c = 0.1\pi$.

**Example 1:** Filter order $N=7$, stop-band attenuation $a_s = 60$ dB. Parameters of for two filters: (i) start-up half-band filter, and (ii) transformed filter, which is obtained from the half-band filter using formulae (5) – (7), are given in Table I.

<p>| TABLE I. PARAMETERS OF THE EXAMPLE 7TH ORDER FILTERS |
|---------------------------------------------|-----------|</p>
<table>
<thead>
<tr>
<th>Filter Order</th>
<th>Half-band filter</th>
<th>Transformed filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_c$</td>
<td>0.5$\pi$</td>
<td>0.1$\pi$</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>0.370608$\pi$</td>
<td>0.066132$\pi$</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>0.629392$\pi$</td>
<td>0.150323$\pi$</td>
</tr>
<tr>
<td>$a_p$</td>
<td>4.34x10^4 dB</td>
<td>4.34x10^4 dB</td>
</tr>
<tr>
<td>$a_c$</td>
<td>60 dB</td>
<td>60 dB</td>
</tr>
<tr>
<td>$a_s$</td>
<td>0.726542528</td>
<td>-0.951056516</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>-0.951056516</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>0.1090105323</td>
<td>0.6022211234</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>0.3849773186</td>
<td>0.75866810520</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>0.76117384230</td>
<td>0.91956148280</td>
</tr>
</tbody>
</table>

**Example 2:** Filter order $N=3$. This example is designed to exhibit the transition band, $\omega_p - \omega_l$, equal to the transition band of Example I. Here the stop-band attenuation is considerably lower and amounts $a_s = 22.3$ dB. Parameters of for two 3rd order filters: (i) start-up half-band filter, and (ii) transformed filter, which is obtained from the half-band filter using formulae (5) – (7), are given in Table II.

<p>| TABLE II. PARAMETERS OF THE EXAMPLE 3RD ORDER FILTERS |
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<td>0.150323$\pi$</td>
</tr>
<tr>
<td>$a_p$</td>
<td>0.025 dB</td>
<td>0.025 dB</td>
</tr>
<tr>
<td>$a_c$</td>
<td>22.2995 dB</td>
<td>22.2995 dB</td>
</tr>
<tr>
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<td>$\beta_l$</td>
<td>0.4999492810</td>
<td>0.813204412</td>
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<td>$\beta_s$</td>
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Example 1: Filter order $N=7$, stop-band attenuation $a_s = 60$ dB. Parameters of for two filters: (i) start-up half-band filter, and (ii) transformed filter, which is obtained from the half-band filter using formulae (5) – (7), are given in Table I.

### Table I. Parameters of the Example 7th Order Filters

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<td>$\beta_i$</td>
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**Example 2:** Filter order $N=3$. This example is designed to exhibit the transition band, $\omega_p - \omega_l$, equal to the transition band of Example I. Here the stop-band attenuation is considerably lower and amounts $a_s = 22.3$ dB. Parameters of for two 3rd order filters: (i) start-up half-band filter, and (ii) transformed filter, which is obtained from the half-band filter using formulae (5) – (7), are given in Table II.

### Table II. Parameters of the Example 3rd Order Filters

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The results of the sensitivity analysis for Example 1 and Example 2 are displayed in Figs 2-6.

Figure 2 depicts the phase sensitivity function \( \partial \phi(\alpha, \omega) / \partial \alpha \) for the first-order section, which is identical for both examples.

\[ \omega = 0.5\pi, \alpha = 0 \]
\[ \omega = 0 \]
\[ \omega = 0.1\pi, \alpha = -0.95106 \]
\[ \omega = 0.7265 \]

Figure 2. Sensitivity of the first-order section

Figs 3 and 4 present the sensitivities for the second-order sections from (3a) and (3b), and Figs 5 and 6 are for the sections (2a) and (2b). The plots in Figs 3-6 show high sensitivity maxima around the 3dB-cutoff frequencies. The sensitivity maximum rapidly increases when the cutoff frequency decreases. Evidently, the section implementing the critical pole pair in the example 7th order filter has very high peak value.

The example 3rd order filter, although with the same transition band as the example 7th order filter, has considerably lower sensitivity maxima.

Inevitably, the fixed-point implementation of equations (5) – (7) introduces the quantization error in constant values of the transformed filter. The sensitivity analysis shows that the quantization error can produce very high derogation in filter characteristics in the implementations of the equations (2a) and (2b) or (3a) and (3b). Better solution can be obtained with tapped cascaded interconnection of two identical all-pass filters described in the next section. The tapped cascaded structure uses the all-pass filters of much lower order, and this way implements sharp filter characteristics with low-sensitivity sections.
3. LOW-SENSITIVITY STRUCTURE, CLASS II COMPLEMENTARY FILTER PAIRS

The low-pass/high-pass complementary filter pair \( H(z) H_c(z) \) can be implemented as a tapped cascaded interconnection of two identical all-pass filters \( A_0(z) \) and \( A_1(z) \) [11] where,

\[
H(z) = \sum_{m=0}^{M} a[m] [A_0(z)/A_1(z)]^m
\]

\[
H_c(z) = \sum_{m=0}^{M} a[m] [A_0(z)/A_1(z)]^m
\]

Here, the tap values \( a[m] \) and \( a'[m] \) are the constants of the minimum phase power-complementary FIR prototype filter pair. FIR prototypes are very low-order FIR filters with the same minimal stop-band attenuation as the overall filter pair \( H(z) H_c(z) \). All-pass filters \( A_0(z) \) and \( A_1(z) \) are the solutions for the all-pass branches in low-pass/high-pass IIR filter pair \( [G_L(z) \ G_{HP}(z)] \) as given in (1a) and (1b). Filters \( G_L(z) \) and \( G_{HP}(z) \) compose the IIR prototype filter pair, which has the same transition band as the overall filter pair \( H(z) H_c(z) \).

On the basis of tapped cascaded interconnection of identical all-pass filters, selective filters with high stop-band attenuation can be achieved by using very low order FIR and IIR prototypes.

Since the crossover frequency of \( H(z) H_c(z) \) is exactly the crossover frequency the IIR prototype pair \( [G_L(z) \ G_{HP}(z)] \), simple tuning formulae (5)-(7) can be used to move the crossover frequency of the start-up half-band filter pair \( H(z) H_c(z) \) to the desired location \( \omega_b \) [5], [6]. Notice that the tap coefficients stay unchanged.

Fig. 7 depicts an efficient structure that implements simultaneously both transfer functions \( H(z) \) and \( H_c(z) \). Formulae for converting \( a[m] \)’s to lattice coefficients \( k_m \) can be found in [10], [12].

![Figure 7. Lattice implementation of power-complementary filter pair](image)

4. APPROXIMATION OF TUNING FORMULAE

Equations (5), (6) and (7) express very simple tuning formulae, which compute the filter constants from the constants of the start-up half-band filter and the desired location of the crossover frequency. However, the equations in the form of (5), (6) and (7) are unsuitable for fixed-point DSP implementation. Therefore, we introduce the modifications and approximations of (5), (6) and (7) that make them more suitable for DSP implementation.

The first problem is to calculate constant \( \alpha \) with sufficient precision. To calculate parameter \( \alpha \), we modify equation (5) to the form

\[
\alpha = -\sin(p/2 - \omega_b),
\]

which is preferred to the expression given in (5) because of the simpler power-series expansion [13], [14].

The second problem is to calculate \( \alpha_i \) and \( \beta_i \). The simplest approach is the power-series expansion of functions (6) and (7). Obviously, increasing the number of terms in the power-series expansion decreases the approximation error, but the approximation error remains considerably large, particularly in the case of equation (6) for \( |\alpha|<0.8 \). As an alternative, the piecewise linear approximation of the curves representing (6) and (7) has been examined [7]. It has been shown that the piecewise linear approximation is a better solution compared to the power-series expansion. The approximation of \( \alpha_i \) is carried out as follows:

\[
\alpha_i = y_i + C_i (\alpha - x_i), \quad x_i < \alpha < x_i',
\]

where

\[
C_i = \frac{y_i - y_{i'}}{x_i' - x_i}, \quad i = 0, 1, \ldots, I_{\text{max}}
\]

and \( x_i', \ y_i', \ i = 0, 1, \ldots, I_{\text{max}} \) are the segment-end points lying on the curve representing equation (6). \( I_{\text{max}} \) is the number of segments.

Equation (7) for calculate \( \beta_i \) values is approximated in a similar manner.

5. SIGNAL PROCESSOR IMPLEMENTATION

5.1. Implementation of IIR Filter Structures Based on Parallel Connection of Two All-Pass Filters

IIR filter structure based on the parallel connection of two all pass filters is suitable for hardware and also for software implementations. For the software implementation on the ADSP-2181 DSP processor, filter routine that is less time/memory consuming can be achieved by grouping the product \( \alpha(1+\beta) \) in the sections (3a) and (3b) into a single coefficient \( \gamma = \alpha(1+\beta) \). This means that the second order sections from (2a) and (2b) are more convenient for the particular signal processor implementation than the second order sections from (3a) and (3b).

In the experiments presented in this paper, the structure of Fig. 1 and Fig. 7 have been implemented on EZ-Kit Lite, which is based on the ADSP-2181 DSP processor [13], [14]. ADSP-2181 is a 33 MIPS, 16-bit fixed-point DSP processor [13]. For Class I, the all-pass filter \( A_0(z) \) is implemented according to (2a) as a cascade of one first-order section and the necessary number of sec-
ond-order sections. The all-pass filter \( A_1(z) \), according to (2b), is composed of the second-order sections only. The Class II is implemented according to the implementation scheme from Fig. 7 where \( A_0(z) \) is the first-order section and \( A_1(z) \) is the second-order section.

5.2. Implementation of Tuning Formulae

Implementation on a fixed-point DSP processor requires additional modification of tuning process described in Section II. Coefficients \( C_i \), given in (14b) need to be scaled to [-1 1] range. Better accuracy is achieved by scaling each segment separately in the following manner:

\[
\alpha_i = s_i \left( y_{\text{local}}^\alpha + C_{\text{local}}^\alpha (t - x_{\alpha_i}^\alpha) \right), \quad x_{\alpha_i}^\alpha < t < x_{\alpha_i+1}^\alpha, \quad (10a)
\]

\[
y_{\text{local}}^\alpha = \frac{1}{s_i} y_i^\alpha, \quad (10b)
\]

\[
C_{\text{local}}^\alpha = \frac{1}{s_i} \left( y_{i+1}^\alpha - y_i^\alpha \right), \quad (10c)
\]

where \( s_i \), \( i = 0, 1, \ldots, I_{\text{max}} \) are power-of-two scaling factors.

Arrays of segment-end points, coefficients \( s_i \) and \( C_{\text{local}}^\alpha \) are calculated in MATLAB and stored in DSP processor memory in a manner suitable for multifunctional instructions.

In the case of the curve specified by equation (7), the approximation requires fewer segments, and no additional scaling is needed.

The procedure of tuning the cut-off frequency is carried out in four steps:

1. Parameter \( \alpha \) is calculated.
2. Parameter \( \alpha_i \) is calculated using the value of \( \alpha \) from step 1.
3. For each second-order section, \( \beta_i \) is calculated.
4. Coefficients of all sections are recalculated and updated.

The approximation error in computing \( \alpha_i \) is of particular significance since the approximate value of \( \alpha_i \) is used to compute \( \beta_i \) values.

Number of DSP instruction cycles needed for the entire algorithm is shown in Table III. Total number of calculations depends on (i) numbers of the second-order sections in \( A_0(z) \) and \( A_1(z) \), \( N_0 \) and \( N_1 \), (ii) number of the segments for piecewise-linear approximations of equation (6), \( N_{\alpha \phi} \), and (iii) number of the segments for piecewise-linear approximations of equation (7), \( N_{\phi \phi} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of cycles needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>31</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>( 5N_{\alpha \phi} + 19 + 3 )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( N_0(4N_{\phi \phi} + 25) + 29 + N_1(4N_{\phi \phi} + 25) + 7 )</td>
</tr>
</tbody>
</table>

Additionally, \( 8N_0 + 19 + 8N_1 + 10 \) cycles are needed for updating the filter coefficients. The presented numbers can be additionally reduced by improving the written code.

6. EXPERIMENTAL RESULTS

The experiments have been performed on the hardware platform consisting of a standard PC and EZ-Kit Lite with the processor ADSP-2181. Frequency responses of the resulting filter pairs have been obtained by direct measurement on the implemented filter examples. A simple DSP procedure for automatic calculation of the filter characteristics from measured data has been developed.

The objective of the experiments was to examine practical aspects of the tuning procedure presented in previous sections. The examples of Class I and Class II filter pairs have been examined.

6.1. Implementation of Class I complementary filter pair

Starting from the 7th order half-band filter of Example 1 specified Table II, the procedure described in Sections 4 and 5 has been applied to changing the location of the cutoff [crossover] frequency. The resulting low-pass filters for several values of \( \omega_k \) are shown in Fig. 8. It can be noticed from Fig. 8 that the implemented tuning procedure produces an error, which increases with the increase of \( 0.5\pi - \omega_k \).

![Frequency response of low-pass filters implemented according to Fig. 1.](image)

\( \omega_k = 0.1\pi, 0.2\pi, 0.3\pi, 0.4\pi, 0.5\pi, 0.6\pi, 0.7\pi, 0.8\pi, 0.9\pi \).

Thick line is for the start-up half-band filter.

Fig. 9 plots the attenuation characteristics for the low-pass/high-pass complementary filter pairs: (i) start-up half-band filter pair, and (ii) transformed filter pair with \( \omega_k = 0.1\pi \). The plots of Fig. 9 illustrate the considerable derogations in the stop-bands produced by the quantization error in the tuning procedure. Evidently, very large steps of \( \omega_k \) produce large errors in stop-band characteristics of higher order Class I filter pairs. Power-complementary property of the implemented filter pairs is verified, and results are displayed in Fig. 10. The im-
plemented filter banks nearly satisfy the perfect reconstruction property of the amplitude response. The error visible in Fig. 10 is caused by the nonlinear effects in signal processing.

![Figure 9](image)

Figure 9. *Class I* complementary filter pairs, Example 1. \(\omega_0 = 0.1\pi\), and \(\omega_0 = 0.5\pi\). Thick line is for the start-up half-band filter pair.

![Figure 10](image)

Figure 10. Power complementary property for *Class I* filter pair implemented in signal processor.

### 6.2. Implementation of *Class II* complementary filter pair

To demonstrate the fixed-point implementation of *Class II* complementary filter pair, we choose an example that exhibits similar amplitude characteristics as the *Class I* filter pair presented in subsection 6.1. The solution for the overall filter pair is based on two half-band filter prototypes:

(i) \(5^{th}\) order minimum phase half-band FIR filter that provides minimum stop-band attenuation \(a_s = 60\) dB. The values of the lattice coefficients for the implementation scheme of Fig. 7 are given in Table IV.

(ii) \(3^{rd}\) order IIR start-up half-band filter that provides the overall filter selectivity. The values of the start-up filter parameters are given in the middle column of Table V.

<table>
<thead>
<tr>
<th>(k_1)</th>
<th>(k_3)</th>
<th>(k_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.384126600</td>
<td>-0.541734240</td>
<td>0.108653760</td>
</tr>
</tbody>
</table>

Starting from the start-up half-band filter pair specified in tables IV and V, and applying tuning formulae (5) - (7) to the \(3^{rd}\) order half-band filter prototype, the crossover frequency of the overall filter pair is moved to the location \(\omega_0 = 0.1\pi\). The right column of Table V displays the parameters for the transformed IIR filter calculated in MATLAB. Plots of Fig. 11 present the amplitude characteristics of both filter pairs of *Class II* in signal processor implementation. As it was expected, the derogation of amplitude characteristics of the *Class II* filter pair is nearly negligible. Very large steps of \(\omega_0\) produce large errors in stop-band characteristics of higher order *Class I* filter pairs, but *Class II* gives the satisfactory solution, see Figs 9 and 11. Power-complementary property of the implemented *Class II* filter pairs is verified, and results are displayed in Fig. 12. The implemented filter banks nearly satisfy the perfect reconstruction property of the amplitude response as in the case of *Class I*, see Fig. 10. The error visible in Figs 10 and 12 are caused by the nonlinear effects in signal processing.

![Figure 11](image)

Figure 11. *Class II* complementary filter pairs, Example 3. \(\omega_0 = 0.1\pi\), and \(\omega_0 = 0.5\pi\). Thick line is for the start-up half-band filter pair.
2181 instruction cycles, or, approximately, 15\% of cycles needed.

The tuning procedure for the Class II filter pair in signal processor implementation for wide range of \(\omega\) values. This Figure shows very good characteristics of implemented filters.

![Figure 12. Power complementary property for Class II filter pair implemented in signal processor.](image)

Fig. 13 displays the low-pass filter characteristics of Class II in signal processor implementation for wide range of \(\omega\) values. This Figure shows very good characteristics of implemented filters.

![Figure 13. Frequency responses of low-pass filters implemented according to Fig. 7.](image)

6.3. Computational efficiency

The tuning procedure for the Class I filter example (\(N=7\), \(N_0=1\), \(N_1=2\), \(N_{ad}=40\), \(N_{dp}=7\)) requires 428 ADSP-2181 instruction cycles, or, approximately, 15\% for 48 kHz sampling frequency at most one sample is missed due to the tuning procedure.

For calculation of each output filter sample (regardless of tuning process), a total of \(8(N_0+1)+14+8N_1+19\) instruction cycles is needed.

The tuning procedure for the Class II filter example (\(N=3\), \(N_0=0\), \(N_1=1\), \(N_{ad}=40\), \(N_{dp}=7\)) requires 322 ADSP-2181 instruction cycles, or, approximately, 10\% of cycles needed for the tuning procedure is slightly reduced comparing to the Class I filter.

For calculation of each output filter sample for the Class II filter structure with \(N_k\) taps total of \(12+N_k[8(N_0+1)+9+8N_1+9+11]\) instruction cycles is needed.

For the presented examples calculation of each output sample is more time-consuming in the case of Class II structure (237 opposed to 65 instruction cycles).

7. CONCLUSION

A new algorithm for signal processor implementation of a low-pass/high-pass IIR digital filter pairs with variable crossover frequency is presented. Two implementation structures are implemented and compared: structure based on the parallel connection of two all-pass filters (Class I filter pairs), and structures based on the tapped cascaded interconnections of two identical all-pass filters (Class II filter pairs). Measured results show that the proposed tuning procedure can be applied when sharp filters are requested, and moreover, the proposed procedure is suitable for tuning the crossover frequency of the filter pair in a wide range. Sensitivity analysis and measured results show the advantage of the solutions based on the tapped cascaded interconnection of two identical all-pass filters (Class II filter pairs).

8. ACKNOWLEDGMENTS

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9. REFERENCES