PERFORMANCE ANALYSIS OF FILTERBANK-BASED MC-CDMA SYSTEMS IN THE PRESENCE OF CARRIER FREQUENCY OFFSET

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ABSTRACT
In this paper, we propose a generalization of the concept of MC-CDMA in which the multicarrier modulation is implemented by means of filterbanks. The aim of the paper is to investigate the performance of the filterbank-based MC-CDMA systems in the presence of a carrier frequency offset. A semi-analytical method to compute the error probability when both generic transmit filters and arbitrary linear receivers are used, considering a multipath fading channel in the downlink, is proposed. The accuracy of the method is evaluated by means of computer simulations, considering several choices of transmit filters, and it is shown to be very accurate. The analytical results obtained with the proposed method allow us to compare in a fast way systems characterized by different filters as well as by different receiver designs.

1 INTRODUCTION
In the field of wireless communications, the combinations of Multi-Carrier (MC) modulation [3] and Code Division Multiple Access (CDMA) [7] have gained considerable interest due to their excellent performance. According to the spreading approach used, these techniques can be classified into two different schemes [4]. In the former, referred to as Multi-Carrier Direct Sequence CDMA (MC-DS-CDMA), spreading is performed in the time-domain, whereas in the latter, named Multi-Carrier CDMA (MC-CDMA), spreading is performed in the frequency-domain [4].

Usually, MC modulation is implemented using an Inverse Discrete Fourier Transform (IDFT) at the transmitter and a DFT at the receiver. DFT-based systems are very attractive from the point of view of complexity, but they are also very sensitive to time and frequency offsets. In particular, an offset of the carrier frequency will cause a loss of orthogonality between subcarriers and this introduces inter-carrier interference (ICI) [9] [8]. This fact is also evident both in MC-DS-CDMA systems [13] and in MC-CDMA systems [12] [14]. In order to combat the degradation due to frequency offsets, a possible approach could be the design of a system that is more robust to this effect than a DFT-based one. A similar system can be easily modeled by means of a filterbank transceiver [5], also known as transmultiplexer [1].

In this paper, we investigate the effect of carrier frequency offset (CFO) on a generalization of the MC-CDMA scheme where MC modulation is performed by means of a transmultiplexer. Since this system can be easily modeled relying on the filterbank theory, we will refer to it as Filterbank-Based MC-CDMA (FB-MC-CDMA). For the sake of simplicity, we will consider only the downlink case, i.e., a system where a base station transmits the signals of all users synchronously to the mobile terminals. In this way, each user receives the data from the other users through the same channel and without time shifts, allowing a more simple receiver implementation. In order to eliminate inter-block interference (IBI), we will restrict our attention to two different approaches based either on a cyclic prefix (CP) as in DFT-based MC systems, or on a zero padding (ZP) appended to a block of data. According to these two different strategies, different combining schemes must be used at the receiver. We propose an unifying approach where the receiver structure is composed of two stages. The first stage performs both the demodulation and the compensation of channel effects, according to the different combining strategies, whereas the last stage performs the correlation with the spreading sequence of the desired user, collecting the energy contributes from each subcarrier.

The effects of CFO on the proposed systems will be analyzed by extending the method for the analytical prediction of error probability proposed in [2] to the case of FB-MC-CDMA. Relying on this method, we will be able to derive very accurate expressions for the error probability in the case of systems based either on different transmit filters or on different IBI suppression techniques and combining schemes. The method in [2] has been derived assuming a time-invariant dispersive channel. However, thanks to a semi-analytical approach, it can be easily extended to the case of a time-varying fading channel.

The accuracy of the proposed method is assessed
evaluating the performance of FB-MC-CDMA systems based on different filterbank transceivers and different combining approaches. The error probability results obtained with the proposed technique can be used to compare the performance of several systems based either on DFT or on more selective filterbanks, and by using arbitrary linear receiver structures. This comparison can be made in a fast way without the use of several time-consuming computer simulations.

2 FILTERBANK-BASED MC-CDMA

The system considered in this study is an extension of the MC-CDMA scheme which uses a filterbank transceiver instead of OFDM to perform MC modulation. In particular, filterbank-based MC-CDMA can be thought as the combination of a frequency-domain spreading with an $M$-subband redundant filterbank transceiver [5] [11].

The scheme of the transmitter referred to the generic $\ell$th user is shown in Fig. 1. The $\ell$th user's symbol $b_\ell(n)$ is spread by a sequence $s_\ell = [s_{\ell,0}, s_{\ell,1}, \ldots, s_{\ell,M-1}]^T$ yielding a vector of $M$ chips. The $M$ chips are then up-sampled by $N$ and fed to the $M$ transmit filters $g_k(n)$. Hence, the outputs of the filters are added to yield the $\ell$th user's transmitted signal $x_\ell(i)$.

The scheme of the receiver for the downlink is shown in Fig. 2. We assume that the base station transmits simultaneously the signals of all active users, so that the $\ell$th user receives the sum of the signals of all users through the same channel. The discrete time channel is modeled as a complex FIR filter $h_n$ of order $L_c$, followed by an additive white Gaussian noise $w(i)$. We assume that the channel changes slowly, so that may be considered as constant if an adequately short time interval is observed. Hence, for each set of $M$ input symbols at the modulator, the $N$ corresponding output samples are fed to a locally time-invariant linear system with a transfer function $C(z) = \sum_{n=0}^{L_c} h_n z^{-n}$. The output of the channel is filtered by the $M$ receive filters $f_k(n)$ and then each filter output is downsampled by $N$ to produce $M$ output streams. Each stream is multiplied by the complex conjugate of the corresponding chip of the $\ell$th user's spreading sequences and then they are combined to produce the decision variable $\hat{b}_\ell(n)$. Usually, we have $N \geq M$, i.e., the system is nonmaximally decimated, with a redundancy $L = N - M$. We will assume that this redundancy is sufficient to avoid inter-symbol interference, i.e., $L \geq L_c - 1$. Moreover, we will assume that the receive filters $f_k(n)$ are designed according to some combining criteria, as it will be explained in the following sections.

2.1 MIMO Model of FB-MC-CDMA

The system previously described can be represented more conveniently as a Multiple-Input Multiple-Output (MIMO) system. If we consider a base station transmitting to a set $I_u$ consisting of $N_u$ active users, we can define the input of the MIMO system as the vector $b_{I_u}(n) = [b_1(n), b_2(n), \ldots, b_{N_u}(n)]^T$. In this way, the spreading of all users can be modeled by means of a $M \times N_u$ spreading matrix $S_{I_u}$ multiplying the vector $b_{I_u}(n)$, where we define

$$S_{I_u} = \begin{bmatrix} s_1 & s_2 & \cdots & s_{N_u} \end{bmatrix}. \tag{1}$$

A MIMO representation of the analysis and synthesis filterbanks is possible by introducing their polyphase matrices $T(z)$ and $R(z)$. If we decompose the transmitting filters $G_k(z)$ into their $N$-th order polyphase components, we have $G_k(z) = \sum_{l=0}^{N-1} G_{k,l}(z) z^{-l}$, where $G_{k,l}(z) = \sum_{n} g_k(nN+l) z^{-n}$ is the $l$-th polyphase component of the $k$-th subband filter $g_k(n)$. The transmitter can be modeled by a $N \times M$ polyphase matrix given by

$$[T(z)]_{m,n} = G_{n,m}(z). \tag{2}$$

The filters at the receiver can be expressed in a similar way as $F_k(z) = z^{N-1} \sum_{l=0}^{N-1} F_{k,l}(z) z^{-l}$, and the resulting $M \times N$ polyphase matrix is given by

$$[R(z)]_{m,n} = F_{m,n}(z). \tag{3}$$

As to the MIMO representation of the channel, by using multirate theory identities [15], it can be described via a pseudo-circulant $N \times N$ matrix. In particular, if we assume $L_c < N$, that is a condition often verified in practical applications, the channel matrix can be ex-
pressed as

$$\mathbf{C}(z) = \begin{bmatrix}
    h_0 & \cdots & 0 & z^{-1}h_{Lc} & \cdots & z^{-1}h_1 \\
    \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
    \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
    0 & \cdots & h_{Lc} & \cdots & h_1 & h_0
\end{bmatrix}.$$  \hfill (4)

Finally, the noise vector process given by $\mathbf{w}(n) = [w(nN), w(nN+1), \ldots, w(nN+N-1)]^T$ is added to the received vector. The decision variable at the $\ell$th user’s receiver is given by multiplying the output vector from the receive filters by the vector $\mathbf{s}_\ell^H$, where the superscript $^H$ indicates the conjugate transposition.

3 RECIIVER DESIGN

In a MC-CDMA system, the energy of the transmitted symbols is spread over all subcarriers. This fact has led most of researchers to consider the combining approach as the most suitable for the receiver, in particular for the single-user case. Since, in the general case, the channel is frequency-selective, each subcarrier undergoes a different attenuation and phase shift. Therefore, the design of the combiner has to face a twofold issue. First, the channel frequency selectivity causes a loss of orthogonality among the spreading sequences, leading to a considerable interference between different users. Secondly, a noticeable frequency diversity is inherent in the different subcarriers, that can lead to a considerable diversity gain for systems that are not fully loaded.

In order to exploit both these characteristics, several combining strategies have been proposed in the literature for the classical MC-CDMA scheme [4]. However, when dealing with the proposed filterbank approach, these combining strategies need to be adapted to the particular framework we employ. In the proposed system, the combining scheme is implemented through a receive filterbank $\mathbf{R}(z)$ followed by the correlation with the vector $\mathbf{s}_\ell^H$. In particular, we integrate the combining criterion into the design of $\mathbf{R}(z)$. We have restricted our attention to two particular forms of the transmit matrix $\mathbf{T}(z)$ that makes use of either a cyclic prefix or a zero padding precoding and to two combining approaches, namely, Maximum Ratio Combining (MRC) and Orthogonality Restoring Combining (ORC).

3.1 Cyclic Prefix

When using cyclic prefix, the transmit matrix can be expressed as

$$\mathbf{T}(z) = \begin{bmatrix}
    \mathbf{G}_0(z) \\
    \mathbf{G}_0(z)
\end{bmatrix},$$  \hfill (5)

where $\mathbf{G}_0(z) = \sum_k \mathbf{G}_0(z)^{-k}$ is an $M \times M$ polynomial matrix that represents a generic maximally decimated synthesis filterbank (i.e. with $N = M$). In the remainder of this paper, except where otherwise specified, we will deal with normalized paraunitary maximally decimated filterbanks [15], for which we have $\mathbf{G}_0(z)\mathbf{G}_0(z)^{-1} = \mathbf{I}_M$, where the notation $\mathbf{A}^H(1/z^*)$.

In DFT-based MC-CDMA, the use of CP is sufficient to implement a one-tap equalizer, so that the combining scheme consist of a single channel-dependent coefficient weighting each subcarrier. When we use a filterbank transceiver, in which transmit filter impulse responses span more than one block, the combining scheme is more complex than in DFT-based systems. Consider the signal at the receiver. Due to the CP, linear convolution is transformed into a circular convolution. It can be shown [10] that if $L \geq L_c - 1$ we have $\mathbf{C}(z)\mathbf{T}(z) = \mathbf{C}_G\mathbf{G}_0(z)$, where $\mathbf{C}_G$ is a circulant matrix. Hence, reminding that a circulant matrix can be diagonalized by means of a DFT, we can express the received signal as

$$\mathbf{y}(n) = \mathbf{W}^H\mathbf{A}_C\mathbf{W} \sum_k \mathbf{G}_0,k\mathbf{s}_\ell b_{Lc}(n - k) + \mathbf{w}(n),$$  \hfill (6)

where $\mathbf{A}_C$ is a diagonal matrix whose elements are the DFT of the channel impulse response and $\mathbf{W}$ is the DFT matrix. Relying on the hypothesis of paraunitary $\mathbf{G}_0(z)$, the decision variable resulting from the three combining schemes can be expressed in a general form as

$$\hat{b}_\ell(n) = \mathbf{s}_\ell^H \sum_k \mathbf{G}_0,H_{0,K-1-k}\mathbf{W}^H \mathbf{D} \mathbf{W} \mathbf{y}(n - k),$$  \hfill (7)

where for MRC and ORC we have $\mathbf{D} = \mathbf{A}_C^H$ and $\mathbf{D} = \mathbf{A}_C^{-1}$, respectively. Hence, the receiver in the CP case can be expressed as

$$\mathbf{R}(z) = \mathbf{G}_0(z)\mathbf{W}^H \mathbf{D} \mathbf{W}.$$  \hfill (8)

3.2 Zero Padding

When using zero padding, we can express the resulting transmitter as

$$\mathbf{T}(z) = \begin{bmatrix}
    \mathbf{G}_0(z) \\
    \mathbf{0}_{L \times M}
\end{bmatrix}.$$  \hfill (9)

As to the receiver filterbank, in the ZP case we can decompose the channel matrix as $\mathbf{C}(z) = [\mathbf{G}_0 \mathbf{C}_1(z)]$, where $\mathbf{C}_0$ is a constant matrix, so that we have $\mathbf{C}(z)\mathbf{T}(z) = \mathbf{C}_0\mathbf{G}_0(z)$ [5]. Hence, we can express the received signal as

$$\mathbf{y}(n) = \mathbf{C}_0 \sum_k \mathbf{G}_0,k\mathbf{s}_\ell b_{Lc}(n - k) + \mathbf{w}(n),$$  \hfill (10)

As in the previous case, we can express the decision variable for the two different combining schemes in a general form as

$$\hat{b}_\ell(n) = \mathbf{s}_\ell^H \sum_k \mathbf{G}_0,H_{0,K-1-k}\mathbf{Q}\mathbf{y}(n - k),$$  \hfill (11)
where for MRC and ORC we have \( Q = C_0^H \) and \( Q = (C_0^H C_0)^{-1} C_0^H \), respectively. In this case, the receiver can be expressed as

\[
R(z) = \tilde{G}_0(z)Q.
\]

\section{Frequency Offset Model}

A carrier frequency offset can be modeled in the discrete time domain by multiplying the received baseband signal by the sequence \( \varphi(z) = \exp(j2\pi f_0 T z) \), where \( \epsilon \) indicates the frequency offset normalized with respect to intercarrier spacing, \( \phi_0 \) is the initial phase and \( M \) is the number of subcarriers. Relying on the MIMO model of Section 2.1, we rewrite the sequence \( \varphi(z) \) as

\[
\varphi(i) = \varphi(nN + m) = e^{j2\pi \epsilon n/M} \varphi(n) \tag{13}
\]

where \( \varphi(n) = e^{j2\pi \epsilon N/M + \phi_0} \). As can be seen, the phase term \( \varphi(i) \) can be represented as the product of two sequences, depending either on \( n \) (the block index) or \( m \) (the sample inside a block). The effect of the first term in equation (13) can be modeled by multiplying the received vector \( y(n) \) by a diagonal matrix \( \Phi \), defined as

\[
[\Phi]_{m,n} = e^{j2\pi \epsilon m/M} \delta_{m,n} \tag{14}
\]

where \( \delta_{m,n} \) is the Kronecker delta. The effect of \( \varphi(z) \) is that all samples of the received modulated symbol experience the same phase shift. We suppose that this phase shift can be perfectly compensated at the receiver. Hence, if we define \( \Psi(z) = R(z)\Phi(z)C(z)T(z) \), the input-output relation of the system affected by frequency offset can be expressed as

\[
\hat{b}_t(n) = s_t^H \sum_k \Psi_{e,k} S_{lu} b_{lu}(n-k) + \epsilon_t(n), \tag{15}
\]

where we consider \( \Psi_{e}(z) = \sum_n \Psi_{e,n} z^{-n} \), whereas \( \epsilon_t(n) \) indicates the noise component at decision device, i.e., \( \epsilon_t(n) = s_t^H \sum_k R_k w(n-k) \) (see Fig. 3). The system suffers from ISI due to the presence of non null matrices \( \Psi_{e,n}, n \neq 0 \), multiplying powers of \( z \). In the absence of ISI, the system suffers from MAI if \( s_t^H \Psi_{e,o} s_m \neq 0 \) for \( \ell \neq m \). To highlight this behavior, let us rewrite (15) as

\[
\hat{b}_t(n) = \delta^{(e)} b_t(n) + \mu^{(e)}(n) + \nu^{(e)}(n) + \epsilon_t(n), \tag{16}
\]

where we define

\[
\delta^{(e)} = s_t^H \Psi_{e,0} s_e, \tag{17}
\]

whereas

\[
\mu^{(e)}(n) = s_t^H \Psi_{e,o} S_{lu-{\ell}} b_{lu-{\ell}}(n), \tag{18}
\]

\[
\nu^{(e)}(n) = s_t^H \sum_{k \neq 0} \Psi_{e,k} S_{lu} b_{lu}(n-k), \tag{19}
\]

model ISI and MAI terms. In particular, \( L_u - \ell \) denotes the set of all active users except the \( \ell \)th user. It is easy to verify that \( \mu^{(e)}(n) \) depends on the symbol transmitted by the other users, while \( \nu^{(e)}(n) \) depends on the symbol transmitted at time indexes different than \( n \) by all users. Hence, if we model the transmitted symbols as a set of WSS white processes with zero mean, then we have that both \( \mu^{(e)}(n) \) and \( \nu^{(e)}(n) \) are additive noise processes statistically independent of \( \hat{b}_t(n) \). The decision variable can be expressed in a shorter form as

\[
\hat{b}_t(n) = \delta^{(e)} b_t(n) + \nu^{(e)}(n), \tag{20}
\]

where the quantity \( \nu^{(e)}(n) = \mu^{(e)}(n) + \nu^{(e)}(n) + \epsilon_t(n) \) can be thought as an equivalent noise.

\subsection{The Effect of \( \delta^{(e)} \) on the Error Probability}

In (20), the effect of a frequency offset is modeled as the combination of the distortion of the constellation of received symbols due to \( \delta^{(e)} \) and an additive noise process. Hence, the error probability \( P_e^{(\ell)} \) relative to the \( \ell \)th user depends on the distortion \( \delta^{(e)} \).

We consider now the error probability conditional to a given distortion \( \delta^{(e)} \) on the transmitted symbols. For the sake of simplicity, we will assume that the transmitted symbols are BPSK modulated, even if the results can be easily generalized to different constellations, by using the same approach as we proposed in [2] for PAM and QAM. Consider the equivalent SNR per bit

\[
\xi_t = \sigma_b^2 / \sigma_{\epsilon_t}^2 \tag{21}
\]

where \( \sigma_b^2 \) and \( \sigma_{\epsilon_t}^2 \) indicate the power of the transmitted bits and the variance of the equivalent noise at the decision device, respectively. Relying on the model in [2], we can express the error probability in the case of BPSK symbols as a function of \( \xi_t \) and \( \delta^{(e)} \) as

\[
P_e^{(\ell)}(\xi_t, \delta^{(e)}) = Q \left( \sqrt{2\xi_t R(\delta^{(e)})} \right). \tag{22}
\]

where we define \( Q(x) = (2\pi)^{-\frac{1}{4}} \int_x^\infty e^{-t^2/2} dt \).

\subsection{Equivalent Noise Computation}

The expression of the error probability is a function of the equivalent SNR per bit denoted as \( \xi_t \) in Section 4.1. Hence, we need to compute \( \sigma_{\epsilon_t}^2 \) as it appears in the definition of \( \xi_t \) (see eq. (21)).
Consider the autocorrelation of the decision variables of the $\ell$th user. From equation (20) and the statistical independence of $b(n)$, $\rho^2_{\ell}(n)$, $\delta^c_{\ell}(n)$ and $c_{\ell}(n)$ we have

$$E \left\{ \hat{b}_i \hat{b}_j^* \right\} = |\delta^{(c)}_{\ell} |^2 \sigma_{b_i}^2 + \sigma_{n_i}^2,$$

(23)

where $\sigma_{n_i}^2 = \sigma_{n_i}^2 + \sigma^2_\delta $ is the variance of the equivalent noise. If we let $E \{ b_i b_j^* \} = \sigma_{n_i}^2 I_N$, and $E \{ w w^H \} = \sigma_{w}^2 I_N$, then we can express the autocorrelation in (23) as

$$E \left\{ \hat{b}_i \hat{b}_j^* \right\} = c_{\ell,i} \sigma_{b_i}^2 + \kappa_{C,\ell} \sigma_{w}^2,$$

(24)

where we define

$$c_{\ell,i} = \frac{1}{2\pi j} \int_{C} s_i^H \Psi_c(z) S_i \bar{S}_i^H \bar{\Psi}_c(z) s_i z^{-1} dz$$

(25)

and

$$\kappa_{C,\ell} = \frac{1}{2\pi j} \int_{C} s_i^H R(z) \bar{R}(z) s_i z^{-1} dz.$$  

(26)

Hence, if we use (24) into (23) we obtain

$$\sigma_{n_i}^2 = \left( c_{\ell,i} - |\delta^{(c)}_{\ell} |^2 \right) \sigma_{b_i}^2 + \kappa_{C,\ell} \sigma_{w}^2.$$  

(27)

Note that the expression of $\sigma_{n_i}^2$ in (27) can be obtained from the knowledge of $\Psi_c(z)$, i.e., from $T(z)$ (the transmitting filterbank), $R(z)$ (the receiving filterbank dependent on the channel $C(z)$ and the equalization method) and $\epsilon$ (the CFO).

Let us gain some insight about the meaning of the term $c_{\ell,i}$. Recalling that $S_i S_i^H = \sum_{i \in I_u} s_i s_i^H$, we can express (25) in a more compact form as

$$c_{\ell,i} = \frac{1}{2\pi j} \int_{C} \sum_{i \in I_u} \rho_{\ell,i}(z) \tilde{\rho}_{\ell,i}(z) z^{-1} dz$$

$$= \sum_{i \in I_u} \int_{\delta}^{\pi} |\rho_{\ell,i}(e^{j\omega})|^2 d\omega$$

(28)

where we define $\rho_{\ell,i}(z) = s_i^H \Psi_c(z) s_i$. From (28) it is evident that each user contributes separately to the noise enhancement in the case of a frequency offset. However, this effect is closely related to the combining strategy at the receiver. In the case of ORC, the terms $|\rho_{\ell,i}(e^{j\omega})|^2$ for $i \neq \ell$ are null in the absence of a frequency offset. Hence, in the presence of CFO we can foresee a performance degradation proportional to the number of active users. Conversely, for an MRC system the terms $|\rho_{\ell,i}(e^{j\omega})|^2$ are different from zero both in the presence and in the absence of CFO. Therefore, we can suppose the in the case of MRC the effect of CFO is not so strictly dependent on the system load.

Finally, the equivalent SNR per bit in (21) can be expressed as

$$\xi_{\ell} = \left[ \left( c_{\ell,i} - |\delta^{(c)}_{\ell} |^2 \right) + \kappa_{C,\ell} \frac{2}{\sigma_{b_i}^2} \right]^{-1}.$$  

(29)

and then it can be used in (22) to evaluate the bit error probability of the system. However, since the values of both $\delta^{(c)}_{\ell}$ and $\xi_{\ell}$ depend on the channel response, in order to derive the error probability for the $\ell$th user in the case of a fading channel we need to compute the expectation of (22) with respect to the probability density function of the channel coefficients $h_n$, i.e.,

$$P_{e(\ell)} = \int P_{e(\ell)}(\xi_{\ell}(h), \delta^{(c)}_{\ell}(h)) p_h(h) dh,$$

(30)

where $h = [h_0, h_1, \ldots, h_{L_c}]^T$. Usually, the expression in (30) can not be resolved in a closed form. Therefore, in this paper it will be evaluated through a semi-analytical approach.

5 SIMULATION RESULTS

The accuracy of the proposed method has been verified by means of Monte Carlo simulations. We have designed
a system with 64 subcarriers, using a 20 MHz bandwidth. The transmitted signal is sampled at Nyquist rate, resulting in a sample period of 50 ns. The system is supposed to operate at a carrier frequency of 5 GHz. We will consider both DFT systems, i.e., \( G_0(z) = W^H \), and systems based on the Modulated Lapped Transform (MLT) [6]. The MLT is a particular cosine-modulated filterbank where the analysis and synthesis filter have length equal to \( 2M \). In particular, it can be efficiently implemented via a fast cosine transform yielding a system complexity comparable to that of DFT systems. The transmit polyphase matrix of the MLT system can be expressed as \( G_0(z) = G_{0,0} + z^{-1}G_{0,1} \) where

\[
\begin{align*}
[G_{0,0}]_{n,m} & = p(n) \gamma(n, m) \\
[G_{0,1}]_{n,m} & = p(n + M) \gamma(n + M, m)
\end{align*}
\]

and \( p(n) \) and \( \gamma(n, m) \) are the filter prototype and the modulation coefficients, respectively, given by

\[
\begin{align*}
p(n) & = \sin \left[ \frac{\pi}{2M} \left( n + \frac{1}{2} \right) \right] \\
\gamma(n, m) & = \sqrt{\frac{2}{M}} \cos \left[ \frac{\pi}{M} \left( m + \frac{1}{2} \right) \left( n + \frac{M + 1}{2} \right) \right].
\end{align*}
\]

Finally, in order to avoid IBI, either CP or ZP techniques are used.

We have verified the correctness of the method in the case of a typical fading channel. We assumed Rayleigh fading statistics and an exponential power delay profile with a rms delay spread of 50 ns. The performance of the proposed FB-MC-CDMA systems has been predicted with our method and compared with the results of the simulations. For each systems, both ORC and MRC have been investigated.

The results for ORC and MRC in the case of the DFT system are shown in Fig. 4 and Fig. 5, respectively, considering \( \epsilon = 0.1 \). Similar results in the case of a MLT system with 64 active users \( (E_b/N_0 = 12 \text{ dB}, \epsilon = \text{FO}) \).
system are shown in Fig. 6 and Fig. 7. In the above figures, points represent simulated results whereas lines indicate the SER performance obtained with the proposed method. As can be seen, in all cases the proposed method appears to be very accurate, since simulation results are almost exactly superimposed on theoretical curves. Only a slight mismatch can be observed for the DFT-ZP system using MRC when we assume the maximum number of active users.

The accuracy of the proposed approach allows us to successfully use it for comparing the performance degradation due to frequency offset on different systems. In Fig. 8 and Fig. 9 the performance of the two DFT-based systems using both MRC and ORC receiver are shown considering different number of active users at an SNR value of 12 dB. We consider three values of $\epsilon$ ranging from $\epsilon = 0$, i.e., in the absence of CFO, to $\epsilon = 0.02$. We point out that in the above figures only SER curves obtained by applying the proposed method are included. In this case, the points laying on the curves do not represent simulation values, but they are only used to better distinguish the different systems.

As can be seen, the performance of a DFT system using the MRC receiver is insensitive to the frequency offset, irrespective of the precoding technique used. Conversely, as to the ORC the two precoding techniques achieve different SER performance in the presence of CFO. In the case of the ZP system, no degradation due to the CFO can be observed, whereas in the case of the CP system we notice a performance loss when the number of active users is increased. We can explain this behavior by taking into consideration the terms $|\rho_{i,}(e^{j\omega})|^2$ in (28) that model the interfering effect of the users. When considering the MRC systems, this interference is present irrespective of the value of the CFO. Hence, we can suppose that a frequency offset does not contribute to the enhancement of this level of interference. On the other hand, in the case of the ORC, interference arise only in the presence of a CFO. Therefore, this explain the performance degradation shown in Fig. 8 for the CP case. As to the ZP system, the fact that no observable degradation is present can be ascribed to the better robustness of the ZP technique to carrier frequency offsets.

Similar results are shown also in Fig. 10 and Fig. 11 considering systems based on the MLT. In this case, the system behavior for different numbers of users is quite different with respect to the DFT systems. This behavior is similar both for CP and ZP, even if the overall performance of the ZP-based system is better than that of the CP-based. In particular, the MRC receiver suffers from a severe performance degradation in the presence of CFO even considering small numbers of active users. The reasons of this behavior are not fully understood and they need further investigation.

6 CONCLUSIONS

We have proposed a semi-analytical method to assess the performance of filterbank-based MC-CDMA in the presence of carrier frequency offset. The theoretical prediction has been verified by means of computer simulations and the proposed method had very accurate results. The proposed method can be successfully used to predict the performance of different FB-MC-CDMA systems and to compare them. These comparisons allows us to choose the solutions that are more effective to combat the effects of carrier frequency offset.

References


