REDUCTION OF THE NUMBER OF COEFFICIENTS IN REED-MULLER HAAR-LIKE SPECTRUM

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ABSTRACT
This paper presents three methods for reduction of the number of nonzero coefficients in the spectra of binary vectors using newly introduced Reed-Muller Haar-like transform. The transform, used to transfer a given truth vector of a switching function into the spectral domain, is based on the Kronecker product of the Reed-Muller and binary parametric Haar-like transforms (BPHT). The BPHT is a signal adapted discrete binary valued transform that may be computed with a fast algorithm having the structure similar to that of classical fast Haar transform. The new hybrid transforms introduced in this paper combine properties of both the Reed-Muller and BPHT transforms due to which they provide reduction of the number of nonzero coefficients in spectral representations of switching functions. Experimental results show that the proposed methods provide reduction in number of spectral coefficients compared to the Reed-Muller transform.

1. INTRODUCTION
Spectral techniques are an efficient tool in computer engineering and logic design. Spectral representations of a switching function are obtained by the decomposition of the function in terms of a set of basis functions. The Haar transform is an example of transforms that have been widely used in digital logic design for many years [1-3]. Many other well-known spectral transforms such as the Reed-Muller, the arithmetic, the Walsh transforms are based on the Kronecker product of the corresponding basic matrix kernels. Such transformations are canonical also when various kernel transformation matrices are mutually different in which case they are called hybrid transforms [3]. Transforms with the Kronecker product representible transform matrices have fast computational algorithms.

Recently, there has been a growing interest in different techniques for reduction of nonzero coefficients in spectral representations of switching functions in order to reduce the hardware requirements. In particular, a considerable amount of work has been related to applications of Reed-Muller (RM) transform [3].

The RM transform has been applied also in many other areas of signal processing such as fault detection, image processing (particularly, compression), testing. For this reason, it was interesting to combine Haar-like transforms with the Reed-Muller transform and apply the resulting transforms to derivation of compact representations of switching functions in the number of coefficients count.

In [6] the parametric Haar-like transform (PHT) was introduced as an input signal-adapted transform that may be adjusted to the input signal characteristics by specifying values for related parameters. For a given signal the transform is designed based on a suitably selected generating vector that appears as the first row in the transform matrix. The transform is constructed in such a way that it may be computed by the fast algorithm having structure similar to that of the fast Haar transform algorithm.

Here we propose spectral methods based on combination of RM and BPHT transforms.

In this paper, we first introduce the binary version of PHT transforms, (BPHT), as a class of transforms over GF(2) designed for each given input signal by taking into account specific features the signal may express. Like PHTs, the BPHTs have also the fast algorithms with the structure similar to that of fast Haar transform. The difference between various BPHTs is in the way of selecting the spectral kernels to define a particular BPHT depending on the features of the signal to be processed. Then, we propose to combine the Reed-Muller transform with the introduced BPHT transforms in order to derive the compact representations of switching functions. We refer to these new hybrid transforms as Reed-Muller Haar-like (RMH-like) transforms, as it will be described in Section 3.1.

We propose three algorithms for reduction the number of nonzero coefficients in Reed-Muller Haar-like spectra of switching functions. These algorithms differ in the way of constructing BPHTs used to define the related RMH-like transforms.

The first method is based on the frequency of occurrence of the most common subvector and the number of nonzero elements (1’s) in the subvectors of a binary truth
vector. This approach utilizes the assumption that the truth vectors of switching functions may contain many repeating patterns or subvectors of a certain length. Then, the subvector having, at the same time, the maximal frequency of occurrence and the maximal number of 1’s is selected for the generating vector in construction of the subvectors of the permuted vector. Thereafter, the RMH-like transform is constructed as the Kronecker product of the Reed-Muller matrix of the corresponding order and the BPHT. The spectral representations in terms of thus defined RMH-like transforms provide the reduction of at about 11% of nonzero coefficients compared to the RM transform.

In the second method, a set of RMH-like transforms is constructed by selecting the generating vectors for BPHT subtransforms from the set of all subvectors of the given length in the truth-vector of the function processed. The experimental results show that by using this method the improvement is at about 27% on the average compared to the Reed-Muller transform.

In the third approach, we extended the search space for the generating vectors from subvectors that appear in the given truth vector to all possible binary subvectors of the specified size.

Then, the RMH-like transform that provides the most compact spectral representation of a given switching function is selected as the optimal transform for the function considered. The experiments show that this method outperforms the RM transform for about 31% on the average.

The paper is organized as follows. The Section 2 presents definitions of the Reed-Muller (RM) and the family of parametric transforms. In Section 3, the binary parametric Haar-like transforms (BPHTs) are defined. In Section 4, the definition of the hybrid RMH-like transform based on combination of Reed-Muller and BPHT transform is presented. Three proposed techniques for reduction of nonzero spectral coefficients by means of the RMH-like spectral transform are described in Section 4. The Section 5 discusses the experiments and results obtained. The section 6 summarizes the features of proposed methods and experimental results.

2. SPECTRAL TRANSFORMS

Spectral transforms are widely used in control and communication theory, signal/image processing, in analysis, synthesis and testing of digital circuits, as well as in compact representation of Boolean functions. In particular, the Reed-Muller and Haar transforms are widely used in these areas [1-4]. The Reed-Muller and binary Haar-like transforms will be used in this paper to define the new classes of spectral transforms for representation of switching functions, and therefore, in this section, we briefly present some basic definitions.

2.1. REED-MULLER TRANSFORM

For an n-variable switching function \( f : \{0,1\}^n \rightarrow \{0,1\} \), the Reed-Muller (RM) transform matrix of order \( n \) is defined by the transform matrix

\[
RM(n) = \begin{bmatrix}
RM(n-1) & 0(n-1) \\
RM(n-1) & RM(n-1)
\end{bmatrix}
\]

RM(1) = \[
\begin{bmatrix}
1 & 0 \\
1 & 1
\end{bmatrix}
\]

where \( 0(n-1) \) is the zero matrix of order \( 2^{n-1} \).

Alternatively, \( RM(n) \) can be expressed as

\[
RM(n) = RM(1) \otimes RM(n-1) = RM(1)^{\otimes n},
\]

where the symbol \( \otimes \) represents the Kronecker product.

Multiplication of the RM transform matrix by the truth vector \( f = [f_0, f_1, ..., f_{n-1}]^T \) of an \( n \)-variable switching function \( f \) results in a new binary vector \( g = [g_0, g_1, ..., g_{n-1}]^T \) of the Reed-Muller spectral coefficients:

\[
g = RM(n) \times f
\]

It is assumed that the calculations are performed over the Galois field GF(2).

Notice that the Reed-Muller transform is a self inverse transform over GF(2), and the function can be reconstructed from its spectrum \( g \) by the same transform matrix.

2.2. FAMILY OF PARAMETRIC TRANSFORMS

In engineering practice, the discrete orthogonal transforms having fast calculation algorithms are mostly used. Examples are the Discrete Fourier, Cosine, Sine, Hadamard, and Haar transforms. Most of these fast calculations, often called fast transforms, can be represented in a unified parametric form involving a set of parameters (see [5,6]). In this way, various transforms of order \( N = 2^m \) can be uniformly represented by the following generalized transform matrix in terms of the product of sparse matrices:

\[
H_N = P^{(m+1)} \prod_{j=1}^{m} H^{(j)} P^{(j)} = \prod_{j=1}^{m} \left( V^{(j)} \otimes 1^{(j)} \right) P^{(j)}
\]

where \( H^{(j)}, j = 1,...,m, \) are \( (N \times N) \) block-diagonal matrices with square \( (2 \times 2) \) blocks \( V^{(j)} \) called spectral kernels or butterflies, located on the main diagonal, and \( P^{(j)}, j = 1,...,m+1, \) are \( (N \times N) \) permutation matrices.

Transforms represented by the transform matrix of the form (1) may be computed by a fast algorithm in \( m \) iterative stages:

\[
x_0 = x; \quad x_j = H^{(j)} \left( P^{(j)} x_{j-1} \right), \quad j = 1,...,m;
\]

\[
y = P^{(m+1)} x_0
\]

where \( x \) is the input signal and \( y \) the spectrum of it.

At the stage \( j = 1,...,m; \) the input vector \( x_{j-1} \) to that stage is first permuted as specified by the permutation matrix \( P^{(j)} \) and then, the resulting vector is multiplied by the block diagonal matrix \( H^{(j)} \), which is equivalent to multiplying the \( (2 \times 2) \) spectral kernels by the corresponding \( (2 \times 1) \) subvectors of the permuted vector.
BINARY PARAMETRIC HAAR-LIKE TRANSFORM

In this section, we introduce the parametric Haar-like transform over the Galois field $GF(2)$, which is therefore called the Binary parametric Haar transform (BPHT).

Besides the different range, the difference between the PHT and BPHT is also in selection of spectral kernels that are selected depending on the values of each pair elements in a predefined generating vector.

We describe here the way of constructing the BPHT for a given generating binary vector with an appropriate selection of the kernels and permutation matrices. In this algorithm we will consider the case when the transform is of order $N=2^n$, since these transforms will be applied to switching functions defined by truth vectors of this size. However, one can consider also the BPHT of an arbitrary order as it was shown for PHT ([6]). The fast algorithm for BPHT may be described as following:

1. Assume that the given generating vector $h$ is the input to a flowgraph of a fast transform (2), which has $m = \log_2 N$ stages, the $j$th stage, $j = 1, \ldots, m$, consisting of $N_j = N/2^j$ butterflies.

   For the first stage:

   2.1. Arbitrarily define the permutation matrix $P^{(1)}$.

      For every pair $[u_{1,s}, v_{1,s}]$ of the generating vector such that $u_{1,s}, v_{1,s}$ is the input to the $s$th butterfly of the first stage of the flowgraph, define a spectral kernel $V^{(1,s)}$ in the following way:

      $$V^{(1,s)} = \begin{cases} 
      \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} & \text{if } u_{ls} = v_{ls} = 1 \\
      \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} & \text{if } u_{ls} = 1, v_{ls} = 0 \text{ or } u_{ls} = v_{ls} = 0
      \end{cases} \quad (3)$$

      for $s = 0, \ldots, N_1 - 1$.

      It can be seen that we are assigning the values of kernel elements 0 or 1 depending on component values of the pair that are input to the $s$th operation of the first stage of the fast Haar transform flowgraph. In other words, for every pair $[u_{1,s}, v_{1,s}]$ we select the corresponding spectral kernel to be the Reed-Muller kernel of order 2, identity matrix of order 2, or the reverse identity matrix of order 2, depending on the values of the corresponding pair of input data. Note that this definition of spectral kernels (as it is in the case of PHT) implies that the second outputs of all butterflies are always zeros which gives the typical structure of fast Haar transform algorithm.

   2.2. Apply the first stage of the flowgraph to the input.
vector $h$ in order to obtain the vector $x_1$.

3. For every stage $j = 2, ..., m$:

3.1. Define the permutation matrix $P^{(j)}$ so that it passes the first outputs of butterflies of the previous stage to the inputs of uppermost butterflies of the current stage. Note that all the $N/2^j$ nonzero outputs of the previous stage will be distributed among $N_j = N/2^j$ butterflies of the current stage.

3.2. Then, again we define the spectral kernels $V^{(j,s)}$ for the pair $[u_{j,s}, v_{j,s}]$ according to the rule (3), where $[u_{j,s}, v_{j,s}]$ are the corresponding two components of the vector $P^{(j)}x_{j-1}$ that are passed to the $s$th operation of the current stage of the flowgraph.

3.3. Apply the $j$th stage of the flowgraph to the vector $x_{j-1}$ to obtain the vector $x_j$.

4. Then, arbitrarily define the set of permutations $P^{(m+1)}$ under the restriction that does not change the position of the first component.

Since the number of nonzero components reduces to a half from stage to stage and since the number of stages is $m = \log_2 N$, only the first output of the flowgraph will be nonzero. The desired transform matrix may be computed as the product of block-diagonal and permutation matrices. However, due to the calculations in GF(2), the transform matrix of the BPHT will not contain the desired predefined vector $h$ in its first row, as it was in the case of PHT [6]. The designed in such a way BPHT matrix will contain the linearly independent rows (columns) over GF(2).

**Example:** Let us consider now an example of designing the BPHT of order 8 with the generating vector $h = [1,0,1,0,1,0,1,0]^T$.

According to (1), the matrix $H_8$ of the desired transform can be written as:

$$H_8 = P^{(4)}H^{(3)}P^{(2)}H^{(2)}P^{(1)}P^{(1)}, \quad (4)$$

where we define $P^{(1)} = P^{(4)} = I_8$. Then, according to the Step 2.2. of algorithm we define

$$H^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

With this matrix we obtain the result of the first stage:

$$x_1 = H^{(1)}h = [1,0,1,0,1,0,1,0]^T.$$

Then, we define the permutation matrix $P^{(2)} = P^{(b)}(8)$ to be the perfect shuffle of order 8. Applying $P^{(2)}$ to $x_1$ results in

$$P^{(2)}x_1 = [1,1,1,0,0,0,0,0]^T.$$ 

Now, according to Step 3.2, we define $H^{(2)}$ as:

$$H^{(2)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \oplus I_4.$$

Applying this matrix to $P^{(2)}x_1$ yields:

$$x_2 = H^{(2)}P^{(2)}x_1 = [1,0,1,0,0,0,0,0]^T.$$

Taking $P^{(3)} = P^{(b1)}(4) \oplus I_4$ and defining

$$H^{(3)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \oplus I_6,$$

we find

$$x_3 = H^{(3)}P^{(3)}x_2 = [1,0,0,0,0,0,0,0]^T.$$

Substituting the defined matrices into the factorization (4) of $H_8$ we obtain the transform matrix:

$$H_8 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

### 3.1. REED-MULLER HAAR-LIKE TRANSFORM

Here we define the RMH-like transform of order $2^n$ in GF(2) which is a hybrid transform that combines the RM and BPHT transforms.

**Definition 1:** The forward Reed-Muller Haar-like (RMH-like) transform of order $2^n$ is defined as

$$T(n,k) = RM(n-k) \otimes H_1(k), \quad (5)$$

where $RM(n-k)$ is the Reed-Muller (RM) transform of order $2^{n-k}$ and $H_1(k)$ is the binary parametric Haar transform (BPHT) matrix of order $2^k$.

**Definition 2:** The inverse Reed-Muller-Haar-like transform (IRMH-like) of order $2^n$ is defined as

$$IT(n,k) = RM(n-k) \otimes InvH_1(k),$$

where the $InvH_1(k)$ is the inverse BPHT.

For different values of parameters $n, k$ one can get different RMH-like transforms. For example, 1. If $k = 1$, then

$$T(n,1) = RM(n-1) \otimes H_1(1) = RM(n),$$

where for $k = 1$, $H_1(1) = RM(1)$.

2. If $k = 2$, then

$$T(n,2) = RM(n-2) \otimes H_1(2);$$

where $RM(n-2)$ is the RM transform of order $2^{n-2}$ and $H_1(2)$ is the BPHT of order $2^2$.

3. If $k = 3$, then
where $\text{RM}(n-3)$ is the RM transform of order $2^{n-3}$ and $H_1(3)$ is the BPHT of order $2^3$.

4. REDUCTION OF THE NUMBER OF NON-ZERO SPECTRAL COEFFICIENTS

In this section we propose three methods that are using the combination of both Reed-Muller and BPHT in order to reduce the number of nonzero spectral coefficients in a resulting RMH-like spectrum. In these methods the spectrum is obtained by applying the transform matrix (5) to an input binary vector. The proposed methods distinguish from each other in the way of constructing the BPHT matrix, that is, how the generating vector for the BPHT is obtained.

4.1. Algorithm 1

For many Boolean functions the truth vectors contain frequently occurring patterns of identical subvectors. Here we exploit this feature in constructing the BPHT matrices.

As it was mentioned in Section 3, the BPHT is designed based on a generating vector. In order to find a generating vector, first of all, we do the following: the original truth vector $f$ of a BF $f$ is subdivided into the non-overlapping subvectors $V_i \in V \subseteq f$, $i = 1,...,2^{n-k}$, of equal lengths $2^k$ ($k$ is some fixed number, $1 < k < n$), where $V = \{V_i\}_{i=1}^{2^{n-k}}$ is the set of all subvectors in the truth vector. Then, for all the subvectors $V_i, i = 1,...,2^{n-k}$, the frequencies of occurrences $Freq(V_i)$ and the number of 1’s $N(V_i)$ for all the subvectors are calculated. The frequency of occurrence $Freq(V_i)$ of the subvector $V_i$ is a measure obtained by counting the number of appearances of the subvector $V_i$ along the truth vector $f$.

Thereafter, for each non-repeated (that is, occurred only once) subvector, from the set of all subvectors of a certain length $2^k$, $1 < k < n$, the total weights are calculated. We define the total weight of a subvector of a truth vector as the product of frequency of occurrence of the subvector and the number of ones contained in the subvector:

$$TW(V_i) = Freq(V_i) \times N(V_i) \quad (6)$$

where $V_i, i = 1,...,2^{n-k}$, is the $i$th subvector of the truth vector. In our method, the generating vector is selected based on the maximum total weight (MTW) criterion defined as

$$MTW = \max_{V_i \in V \subseteq f} \{TW(V_i)\}. \quad (7)$$

Let us denote by $V^*$ the subvector that corresponds to the maximum value of $TW(V_i)$ in (6). The next step in the algorithm is to synthesize the RMH-like transform $T(n,k,V^*)$ with the generating vector $V^*$. Then, the truth vector of a switching function is transferred into the spectral domain by the synthesized RMH-like transform of order $n$:

$$g = T(n,k,V^*) \times f \quad (8)$$

where $f$ and $g$ are the truth vectors of switching function and vector of spectral coefficients in RMH-like domain, correspondingly.

4.2. Algorithm 2

The main difference between this approach and the previous one is in a way of constructing the BPHT matrix.

Like in the first algorithm, we also split the truth vector of the function to be processed into the subvectors $V_i \in V \subseteq f$, $i = 1,...,2^{n-k}$, of equal sizes $2^k$. Then, the set of RMH-like transforms $T(n,k,V_i), i = 1,...,2^{n-k}$, are calculated, where the BPHTs are constructed based on the generating vectors $V_i, i = 1,...,2^{n-k}$, that are all the subvectors of length $2^k$ taken from the input switching function. Then, the RMH-like transform matrices of order $2^n$ are applied to the truth vector of the given switching function to get the spectral representations. Thereafter, the best optimal transform, out of $2^{n-k}$ possible RMH-like transforms, is selected as the transform that requires the minimum number of nonzero elements in the corresponding RMH-like spectrum.

4.3. Algorithm 3

In this algorithm, unlike the Algorithm 2, we extend the search space for the generating subvectors and instead of subvectors appearing in the truth vector, we consider all $2^{2^k}$ possible binary subvectors of order $2^k$, where $k$ is a fixed number and $1 < k < n$. Each of these subvectors is used as the generating vector to synthesize a BPHT of order $2^k$ that participates in RMH-like transforms according to (5). After that, we synthesize the set of $2^{2^k}$ RMH-like transforms and calculate the corresponding RMH-like spectra. Then, the RMH-like transform that gives the minimal number of nonzero coefficients in the spectrum is selected as the optimal representation for the function considered.

5. EXPERIMENTS AND DISCUSSION

The experiments were carried out over 18 different benchmark functions with different number of input variables ($n = 8,9,10,14$) and then the results were averaged. All the truth vectors are splitted into the subvectors of length 8.

The Table 1 shows the number of nonzero spectral coefficients for benchmark functions in the RM (column 4) and RMH-like spectra determined by using the three algorithms (columns 5,6,7) proposed above.

The number of input variables for each switching benchmark function is given in the second column labeled by in.

The number of ones of switching functions in original domain is given in the third column denoted by #1.
Table 1  Number of nonzero coefficients in Reed-Muller spectrum (RM) and in the RMH-like spectrums of the proposed algorithms.

<table>
<thead>
<tr>
<th>BF</th>
<th>in</th>
<th>#1</th>
<th>RM</th>
<th>ALG1</th>
<th>ALG2</th>
<th>ALG3</th>
</tr>
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<tbody>
<tr>
<td>clip0</td>
<td>9</td>
<td>256</td>
<td>86</td>
<td>88</td>
<td>82</td>
<td>80</td>
</tr>
<tr>
<td>clip1</td>
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<td>256</td>
<td>86</td>
<td>78</td>
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<td>63</td>
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<td>78</td>
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</tr>
<tr>
<td>clip4</td>
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<td>113</td>
<td>104</td>
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<tr>
<td>9sym</td>
<td>4</td>
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<td>272</td>
<td>161</td>
<td>149</td>
<td>149</td>
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<td>rnd10</td>
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<td>871</td>
<td>398</td>
<td>338</td>
<td>316</td>
<td>310</td>
</tr>
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<td>rd84_1</td>
<td>8</td>
<td>121</td>
<td>137</td>
<td>70</td>
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<td>39</td>
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<td>ALU4_0</td>
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<td>ALU4_1</td>
<td>14</td>
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<td>13</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>ALU4_2</td>
<td>14</td>
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<td>857</td>
<td>857</td>
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<td>2657</td>
<td>2304</td>
<td>1938</td>
<td>1938</td>
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<tr>
<td>#1 in spectrum</td>
<td>65238</td>
<td>8478</td>
<td>7568</td>
<td>6173</td>
<td>5828</td>
<td></td>
</tr>
<tr>
<td>average #1 in spectrum</td>
<td>471</td>
<td>420</td>
<td>343</td>
<td>324</td>
<td></td>
<td></td>
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<tr>
<td>improvement compared to RM (%)</td>
<td>10.7</td>
<td>27.2</td>
<td>31.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The row denoted by average #1 in spectrum shows the average numbers of nonzero spectral coefficients per function for the Reed-Muller (RM) and RMH-like spectrums determined by the Algorithms 1,2,3, respectively. It follows from the table that the Algorithm 1 results on the average in about 51 less nonzero coefficients in the spectrum than the RM transform. This means 10.7% of improvement over the RM spectrum. The Algorithm 1 was also tested for the case of splitting the truth vectors into subvectors of length 4 and 16. Results show that the Algorithm 1 in the case of splitting the truth vectors into subvectors of length 4 on the average increases the number of nonzero spectral coefficients. It counts on average about 520 nonzero coefficients in the RMH-like spectra versus 471 nonzero coefficients in the RM spectra. For the Algorithm 1 in the case of splitting into subvectors of length 16, the average number of nonzero elements is even less, that is, 392.4 (16.7% improvement over RM). However, this result was not included into the table because of considerable overhead information cost point of view.

In 6th column of the table (ALG2), the results obtained by applying the Algorithm 2 are given. The experimental results show that in most of the cases the Algorithm 2 reduces the average number of nonzero spectral coefficients and it is about 27.2% better than the Reed-Muller transform.

Meanwhile, the Algorithm 3 decreases the number of nonzero spectral coefficients not only in RM spectrum but it also outperforms both the Algorithm 1 and Algorithm 2. Its performance is on the average at about 31.3% better than the RM transform.

6. CONCLUSIONS

The Binary parametric Haar-like transform (BPHT) is defined as a signal adapted transform over GF(2). For a switching given function the BPHT is defined by selecting a generating vector. This transform is combined with the Reed-Muller transform to produce a hybrid Reed-Muller Haar-like transform which shares good properties of both Reed-Muller and Haar-like transforms. Three methods to select the generating vectors for BPHTs are proposed, resulting in RMH-like spectra with different number of nonzero coefficients. The proposed methods are compared with the Reed-Muller transform performance. The experiments carried out over standard benchmark functions show that the proposed methods using RMH-like transforms require fewer number of nonzero elements to represent a switching function compared to the number of nonzero spectral coefficients in the Reed-Muller spectrum.

7. REFERENCES